Debunking the myth of a "flat Earth"!

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Abstract. Even if we weren't able to view the Earth from outer space, an experiment performed on the surface of the Earth would reveal that this surface is not flat, since the axioms of Euclidean geometry do not apply on it.

According to a "scientific" myth, the surface of the Earth may be flat instead of curved**.** Of course, it only takes a look at our planet from outer space to discredit this belief. But, let us suppose that we have no means to travel to outer space and, moreover, we do not trust the pictures of the Earth taken from there. How can we test the geometry of the surface of the Earth (particularly, whether it is flat or curved) without ever leaving the Earth?

In this note we will describe an experiment that would provide an answer to this question. Before we proceed, however, we need to review some concepts from geometry.

A *flat surface* is a plane surface in three-dimensional space [we do not consider *intrinsically flat* surfaces, such as the surface of a cylinder (this concept will be explained later)]. The plane is a two-dimensional *Euclidean* space possessing the following properties (among others):

* The shortest-distance path connecting any two points on the plane belongs to the (unique) straight line passing from these points. In general, shortest-distance curves on a surface are called *geodesics* of this surface. Thus, the geodesics of a plane are straight lines.

* Two geodesics (straight lines) that are initially parallel to each other remain parallel forever; hence they never intersect.

* The sum of the angles of a geodesic triangle[1] is 180^o.

* The Pythagorean theorem is valid: If ABC is a right triangle, where the right angle is at A, then

$$(AB)^2 + (AC)^2 = (BC)^2 \tag{1}$$

Here are now the two stages of our experiment:

1. Choose any two points A and B on the surface of the Earth. Identify the shortest possible path on the surface from A to B (this path will be part of a geodesic curve of this surface) and draw the line AB that connects these points. If we assume that the Earth is flat, this line must be a straight line. Now, place two friends of yours at the locations A and B and ask them to start moving in a direction normal to the line AB at their respective locations, hence parallel to each other. In a Euclidean (flat) space, parallel lines do not

[1] A geodesic triangle is a triangle formed by geodesic-line segments. On a plane this is just the ordinary triangle of geometry.

intersect and thus your friends will never meet on the Earth. Each of them will go as far as the edge of the plane surface and then fall off to outer space!

But if the Earth is, say, a sphere, the surface of which is a two-dimensional *non-Euclidean* space, the Euclidean axioms will not be valid on this surface. Geodesics on the sphere are *great circles*. The line *AB* of minimum distance will thus belong to a great circle, e.g. the Equator, and the paths of your friends will follow the meridians passing from *A* and *B*, both of which are geodesic lines (i.e., great circles) *normal* to the Equator at the respective locations *A* and *B* (see Fig.1). Although initially parallel, these paths will eventually intersect at the North Pole (point *P* in Fig.1) and that's where your two friends will meet again. Moreover, the geodesic triangle *ABP* on the sphere will possess two (!) right angles (at *A* and *B*) and hence the sum of its angles will exceed 180^{o}.

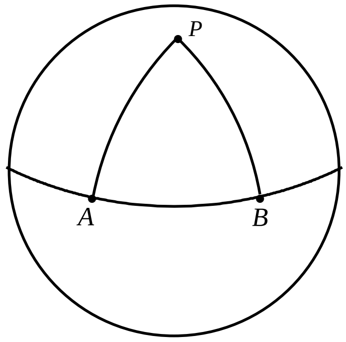

Fig. 1

2. Consider next a huge right triangle *ABC* on the surface of the Earth (with right angle at *A*), making sure that the paths connecting the points *A*, *B*, *C* are the shortest possible (i.e., are parts of corresponding geodesics; see Fig.2). If the Earth is flat, the Pythagorean theorem (1) must be valid, which will not be the case if the Earth is spherical, since the surface of a sphere is a non-Euclidean space. In the latter case, in place of the equality (1) we will have the inequality

$$(AB)^2 + (AC)^2 > (BC)^2 \qquad (2)$$

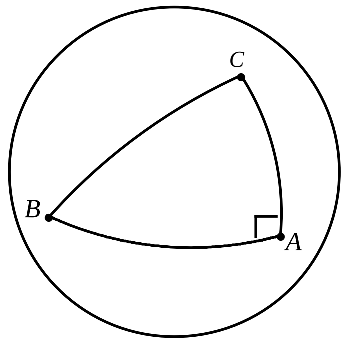

Fig. 2

Let us check this by means of an example: If we place A at Thessalonica, Greece, and if at B and C we put Madrid, Spain and Riga, Latvia, respectively, we form (approximately) a right triangle with right angle at A (as before) and sides equal to

$$AB \text{ (Thessalonica – Madrid)} = 2243 \; km \, ,$$

$$AC \text{ (Thessalonica – Riga)} = 1816 \; km \, ,$$

$$BC \text{ (Madrid – Riga)} = 2712 \; km \, ,$$

(as measured *on* the Earth), so that

$$(AB)^2 + (AC)^2 = 8{,}328{,}905 \; km^2 \quad \text{while} \quad (BC)^2 = 7{,}354{,}944 \; km^2 .$$

Thus the inequality (2) seems to be valid, which would not be the case if the Earth were flat.

Conclusion: Even if we weren't able to view the Earth from afar, an experiment on the surface of the Earth would reveal that this surface is not flat since the axioms of Euclidean geometry do not apply on it.

Note for the more advanced student

In an n-dimensional Euclidean space it is possible to define orthogonal (Cartesian) coordinates x^k (k=1,2,…,n) such that the elementary (infinitesimal) distance ds between any two neighboring points is given by the expression

$$ds^2 = \Sigma_k (dx^k)^2 \tag{3}$$

where Σ_k denotes a sum for k=1,2,…,n. [Relation (3) expresses the generalized Pythagorean theorem in n dimensions.]

For example, in 3-dimensional Euclidean space we define the Cartesian system $(x^1, x^2, x^3) \equiv (x, y, z)$ and express ds in the form

$$ds^2 = (dx)^2 + (dy)^2 + (dz)^2 \, .$$

On the contrary, is not possible to define such a system of coordinates on the surface of a sphere. This explains why we cannot develop a section of a spherical surface on a plane without elastically stretching it. We say that a spherical surface is a *genuinely curved space*.

This is not the case with a *cylindrical* surface. Indeed, one may cut this surface and develop it on a plane without any stretching being necessary. Moreover, on such a surface it *is* possible to define a system of Cartesian coordinates. We say that a cylindrical surface is *intrinsically flat*.