Gravitation and spacetime geometry: A conceptual view

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Some basic ideas of gravitational theory (both classical and relativistic) are discussed with minimum use of mathematical formalism. In particular, the effect of gravity on the geometry of spacetime is explained.

1. Introduction

In a recent book on Special Relativity (SR) [1] we have seen that, in the context of this theory, spacetime is regarded as a *flat* pseudo-Euclidean (or *Lorentzian*) space. It was mentioned, however, that, according to General Relativity (GR), spacetime becomes *curved* in the presence of gravity. This means that gravity is intimately related to the intrinsic geometric characteristics of spacetime.

The connection between gravity (a physical concept) and geometry (a mathematical one) is not an easy thing to explain without recourse to some advanced mathematics. In this article, however, we will attempt to describe some fundamental ideas of GR at a mostly conceptual level. For a fuller appreciation of the theory the reader is referred to the many excellent textbooks available in the literature (for a partial list, see, e.g., [2-9], although many other great books exist).

At the heart of GR is the *principle of equivalence* of inertial and gravitational mass, by which principle gravity is seen to be locally equivalent to acceleration. On the other hand, acceleration is intimately related with non-inertial frames of reference, thus with non-Lorentzian systems of spacetime coordinates with non-constant metric, thus with potentially curved spacetime. In rough terms, this line of reasoning explains the connection of gravity with geometry.

Let us begin our discussion with an overview of the basic differences between SR and GR. These ideas will be refined in subsequent sections.

2. Special vs. General Relativity

According to both SR and GR it is impossible to define absolute velocity. However, in SR the acceleration continues to have an absolute meaning, in the sense that the physical condition of vanishing (equivalently, non-vanishing) acceleration is Lorentz-invariant. Moreover, in SR we assume the existence of an infinite set of *inertial frames of reference*, all being equivalent to each other with regard to expressing physical laws. As defined, an inertial frame has the property that, relative to it, a *free* particle (a particle subject to no net interaction) moves uniformly (with constant velocity, hence no acceleration). The *Lorentz transformation*, relating inertial frames, guarantees that zero (nonzero) acceleration in one frame means zero (nonzero) acceleration in any other frame [1].

No such preferred class of frames of reference exists in GR, where gravity is taken into account. Indeed, no inertial observer (regarded as a “free body”) may exist in the Universe, given that everything possessing mass and/or energy is subject to the universal gravitational interaction.

Geometrically, the spacetime of SR is a *flat* space. It thus allows for a class of Lorentzian coordinate systems which correspond to inertial frames and in which the metric tensor has the standard constant matrix representation $\eta_{\mu\nu} = \pm\,\delta_{\mu\nu}$ or, in explicit diagonal-matrix form, $\eta = diag\,(1,–1,–1,–1)$. These coordinate systems, in turn, admit a special kind of *straight worldlines* representing inertial motions of free particles.

In GR the flat spacetime is replaced by a 4-dimensional *curved* Riemannian space in which there are no preferred coordinate systems. Specifically, there are no *global* Lorentzian frames admitting a globally constant metric. However, the *local flatness* of spacetime allows us to define Lorentzian coordinates in a sufficiently small neighborhood of any spacetime point, as well as a locally constant metric equal to the flat-space metric $\eta_{\mu\nu}$. To see the physical meaning of these geometric ideas we must first state the *equivalence principle*.

3. The principle of equivalence

The principle of equivalence is at the heart of gravitational theory, both classical and relativistic. In this section we discuss the principle within the context of Newtonian gravitational theory. Our conclusions, however, can be readily extended to the relativistic (high-speed) domain.

First, some necessary definitions:

Gravitational mass (m_G): the “charge” of the gravitational interaction; that is, the physical entity that acts as the source of the interaction while at the same time being subject to it.

Inertial mass (m_I): the physical quantity entering Newton’s law, $F = m_I\, a$.

Freely falling body: a moving body subject to no forces other than gravity.

Inertial observer (or frame) inside a gravitational field: an observer (or frame) subject to non-gravitational forces that exactly counterbalance the gravitational force, thus preventing free fall of the observer (or frame). Example: a stationary observer on the surface of the Earth (assuming the latter to be an “inertial” frame).

Consider now a particle inside a gravitational field $\vec{g}\,(\vec{r})$, where $\vec{r}$ is the position vector relative to the origin of an inertial frame. The gravitational force on the particle at a given point is $\vec{F}_G(\vec{r}) = m_G \vec{g}(\vec{r})$, where m_G is the gravitational mass of the particle. Assuming that no other forces (of non-gravitational origin) act on the particle, we have that, by Newton’s law, $\vec{F}_G(\vec{r}) = m_I\, \vec{a}(\vec{r})$, where m_I is the inertial mass and $\vec{a}(\vec{r})$ is the local acceleration of the particle. Hence,

$$m_I\, \vec{a}\,(\vec{r}) = m_G\, \vec{g}\,(\vec{r})\,.$$

By postulating that

$$m_I = m_G \tag{1}$$

(an assumption supported by ample experimental evidence; see, e.g., [8]) we have:

$$\vec{a}\,(\vec{r}) = \vec{g}\,(\vec{r}) \tag{2}$$

This leads to an important conclusion:

> *All bodies at the same place in a gravitational field experience the same acceleration, equal to the field strength, regardless of their mass or internal composition.*

Relation (1) identifying gravitational with inertial mass expresses the *principle of equivalence* and constitutes one of the most important principles of gravitational theory. Relation (2) then implies an equivalence between *gravity* and *acceleration*. Let us see the physical implications of this equivalence.

An *inertial* observer whose local laboratory is placed in a gravitational field will observe that all bodies that are subject to no other forces except gravity (i.e., all *freely falling* bodies) experience a common acceleration in the laboratory. On the other hand, an *accelerated* observer (relative to an inertial frame) *away from any gravitational field* will notice that, relative to her frame, all *free* particles have a common acceleration. Thus, by assuming the validity of the equivalence principle (1), we come to the conclusion that a local observer cannot tell whether his frame is an inertial one placed inside a gravitational field, or an accelerating one away from any gravitational field. One might say that "gravity is traded for acceleration", as suggested by Eq. (2).

Strictly speaking, the equality (2) concerns a particular point in a gravitational field. If the field varies significantly from one point to another inside the observer's laboratory, then identical test particles will experience different accelerations at different points, which will indicate the presence of an inhomogeneous gravitational field. We will assume that the field varies little (i.e., is almost homogeneous) inside the observer's laboratory. Equivalently, the lab's dimensions are small compared to the distance over which the field varies appreciably so that *tidal effects* [8] become noticeable.

Assume now that the observer's laboratory and all particles inside it are in *free fall* under the sole action of gravity. Then, relative to an inertial frame, the lab and the particles will have a common acceleration equal to the field strength, hence the particles will not accelerate relative to the observer. The latter may thus assume that her lab is an "inertial" frame away from gravity, and all particles in the lab are "free" particles. *In a freely falling frame the effects of gravity are eliminated!* Again, the dimensions of the lab must be sufficiently small compared to the distance of appreciable change of the gravitational field.

We say that *a freely falling laboratory looks locally like an inertial frame of reference*. Of course, no *global* inertial-looking frame can be defined by any freely falling lab in an *inhomogeneous* gravitational field (the case of a globally homogeneous field is purely theoretical).

4. Curved spacetime

We now pass to the relativistic point of view. According to SR, an inertial Lorentz frame is equivalent to a system of spacetime coordinates

$$x^\mu \equiv (x^0, x^1, x^2, x^3) \equiv (ct, x, y, z)$$

with metric element (infinitesimal spacetime interval)

$$\begin{aligned} ds^2 = \eta_{\mu\nu}\, dx^\mu dx^\nu &= (dx^0)^2 - (dx^1)^2 - (dx^2)^2 - (dx^3)^2 \\ &= c^2 dt^2 - dx^2 - dy^2 - dz^2 \end{aligned} \tag{3}$$

where the standard summation convention of summing from 0 to 3 over repeated up and down indices has been used, and where $\eta_{\mu\nu}$ is an element of the constant matrix (playing the role of metric tensor in *Minkowski space*)

$$\eta \equiv [\eta_{\mu\nu}] = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} = diag\,(1,-1,-1,-1) \tag{4}$$

The existence of a *global* (pseudo-)Euclidean system of coordinates and a *globally* constant metric tensor $\eta_{\mu\nu}$ is an indication that the spacetime of SR is *flat*.

Coordinate transformations ($x^\mu \to x^{\mu\,\prime}$) between Lorentz frames are linear transformations of the form

$$x^{\mu\,\prime} = \Lambda^\mu{}_\nu\, x^\nu \tag{5}$$

and are such that the metric element ds^2 is left invariant:

$$ds^2 = \eta_{\mu\nu}\, dx^\mu dx^\nu = \eta_{\mu\nu}\, dx^{\mu\,\prime} dx^{\nu\,\prime} \tag{6}$$

It follows that the matrix $\Lambda \equiv [\Lambda^\mu{}_\nu]$ satisfies the relation

$$\Lambda^t \eta\, \Lambda = \eta \quad \Leftrightarrow \quad \Lambda^\mu{}_\lambda\, \eta_{\mu\nu}\, \Lambda^\nu{}_\rho = \eta_{\lambda\rho} \tag{7}$$

(where Λ^t is the transpose of the matrix Λ). The coordinate transformations defined by Eqs. (5)-(7) constitute the *Lorentz transformations* of SR (see, e.g., [1]).

Transforming from an inertial to an *accelerating* (thus non-inertial) frame is equivalent to transforming to general (“curvilinear”) coordinates $\bar{x}^\mu$ that are functions of the x^λ: $\bar{x}^\mu \equiv \bar{x}^\mu(x^\lambda)$, where by x^λ we collectively denote the $(x^0,...,x^3)$. In terms of the new coordinates the spacetime interval is expressed as

$$ds^2 \equiv \eta_{\mu\nu}\, dx^\mu dx^\nu = g_{\mu\nu}(\bar{x}^\lambda)\, d\bar{x}^\mu d\bar{x}^\nu \tag{8}$$

A transformation of this kind (as, more generally, any transformation relating non-inertial frames) is generally *nonlinear* (see Appendix). In flat spacetime a coordinate transformation may always transform a general metric $g_{\mu\nu}(\bar{x}^\lambda)$ back to the standard constant metric $\eta_{\mu\nu}$ of SR.

In the presence of gravity, only an approximately inertial frame may be *locally* defined in a sufficiently small neighborhood of any spacetime point by considering a freely falling frame, as discussed in Sec. 3. In geometrical terms, the flat spacetime of SR where gravity is absent is analogous to a Euclidean (here, pseudo-Euclidean) space in which a *global* Cartesian (here, Lorentzian) coordinate system and an associated constant metric can be defined. No global inertial frame and Lorentzian coordinate system can be defined, however, in spacetime when gravity is present. Accordingly, no global coordinate transformation from general to Lorentzian coordinates exists that will transform a coordinate-dependent metric $g_{\mu\nu}(\bar{x}^\lambda)$ to the constant metric $\eta_{\mu\nu}$ of SR at *all* spacetime points. This is analogous to the absence of a global Cartesian system of coordinates and a globally constant metric in an intrinsically *curved* space. We thus conclude that, *in the presence of gravity, spacetime must be treated as a curved space* that is only *locally flat*. (Cut a tiny patch off a big spherical surface. It will certainly look almost flat!)

5. Geodesics in curved spacetime

As follows from the principle of equivalence, Eq. (1), the trajectory of a freely falling particle (subject only to gravity) is independent of the mass or the internal composition of the particle and is related to the gravitational field itself. This field is determined by the distribution of matter and energy in space and time and, as we have seen, produces an *intrinsic curvature* of spacetime and thus determines the *geometry* of spacetime. It may thus be said that motion under gravity is related to the properties of spacetime itself.

Any freely falling particle follows a *geodesic* [8] of curved spacetime. This may be defined as a *"straightest possible path"* in spacetime, i.e., a curve whose direction is unchanging as one moves along it (in geometrical terms, the tangent vector to the curve is parallel-transported along the curve). Equivalently, given two spacetime points *A* and *B*, a geodesic connecting *A* with *B* is the unique path of extremal length[1] from *A* to *B*.

In particular, even light follows a geodesic of curved spacetime in a gravitational field. Indeed, since light carries energy (consisting of photons) it must also carry inertia, according to the equivalence between these two physical properties. Thus, by the equivalence principle of gravitation, light carries an effective "gravitational mass" and is thus affected by the distribution of matter and energy, hence by the geometry of spacetime.

In flat spacetime (i.e., away from gravity) geodesics are *straight* lines and correspond to inertial motions of free particles and propagation of light rays.

6. Local inertial frames

We have seen that, as a consequence of the principle of equivalence, it is possible to *locally* remove the effects of gravity. Moreover, a gravitational field is *locally* equivalent to an accelerated frame of reference. Let us take a closer look at these issues by first treating the problem in the framework of classical Newtonian theory.

Consider a particle of inertial mass m_I and gravitational mass m_G , placed inside a *uniform* gravitational field $\vec{g} = const.$ The gravitational force on the particle is equal to $m_G \vec{g}$. The particle is also subject to a *non-gravitational* force of the form $\vec{F}'(\vec{R})$, where $\vec{R}$ is the position vector of the particle relative to the physical agent responsible for this force. The motion of the particle relative to an *inertial* frame of reference is determined by Newton's law:

$$m_I \frac{d^2\vec{r}}{dt^2} = m_G \vec{g} + \vec{F}'(\vec{R}) \ ,$$

where $\vec{r}$ is the position vector of the particle relative to the origin of the inertial frame. By postulating that $m_I = m_G \equiv m$ (equivalence principle) we have:

$$m \frac{d^2\vec{r}}{dt^2} = m\vec{g} + \vec{F}'(\vec{R}) \tag{9}$$

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[1] Literally: maximum proper time [8].

We now shift to a *freely falling* reference frame inside the uniform gravitational field $\vec{g}$. According to Eq. (2), the frame (as any freely falling object) has acceleration equal to the field strength at any point. Thus the freely falling frame has constant acceleration $\vec{g}$ relative to the inertial frame, executing uniformly accelerated motion.

Let $\vec{r}'$ be the position vector of the particle *m* relative to the origin of the falling frame. The acceleration of *m* relative to this frame is equal to

$$\frac{d^2\vec{r}'}{dt^2}=\frac{d^2\vec{r}}{dt^2}-\vec{g}\ , \quad \text{so that} \quad \frac{d^2\vec{r}}{dt^2}=\frac{d^2\vec{r}'}{dt^2}+\vec{g}\ .$$

Combining this with (9) and eliminating $m\vec{g}$, we find:

$$m\frac{d^2\vec{r}'}{dt^2}=\vec{F}'(\vec{R}) \tag{10}$$

We observe that *the gravitational field has been "washed off" in the freely falling frame*.

Now, a particle *m* that is subject to no forces other than gravity will also be in free fall with acceleration $\vec{g}$ relative to the inertial frame, thus it will have no acceleration relative to the falling frame. Indeed, Eq. (10) with $\vec{F}'(\vec{R})=0$ yields

$$\frac{d^2\vec{r}'}{dt^2}=0\ ,$$

which means that the particle moves *uniformly* relative to the falling frame, thus describes a straight-line trajectory in that frame. We conclude that, from the point of view of an observer in the falling frame, the particle *m* appears to be a "free" particle moving with constant velocity inside an apparently "inertial" frame *away* from any gravitational field.

Moreover, *in the freely falling frame even light propagates in straight lines*, like any massive freely-falling particle. (As remarked previously, the consequences of the equivalence principle also affect light given that it carries energy, thus inertia, thus an effective gravitational "mass".) However, the path of light will be *curved* relative to an *inertial* frame located somewhere inside the gravitational field. In other words, an inertial observer will notice a *bending of light* due to the gravitational field (see Appendix).

As follows from the above discussion, when performing non-gravitational experiments in a laboratory we cannot tell whether our lab is a truly inertial frame, away from any gravitational field, or a freely falling frame inside a uniform gravitational field.

More generally, we can *locally* remove the effects of a gravitational field even if it is *non-uniform* by choosing a freely falling frame, provided that the size of this frame is small compared to the characteristic length over which the gravitational field varies appreciably.

Generalizing to relativity we may say that, even in the presence of gravity, the curved spacetime is *locally* almost indistinguishable from flat Minkowski space and the laws of Physics are seen to be the usual special relativistic laws in the absence of gravity. Hence, *at each point* of spacetime (or, in a small neighborhood of such a point) we can always define a system of Lorentzian coordinates corresponding to a freely falling frame. This inertial-looking frame is called *local inertial frame*. Of course, a *global* inertial frame and a *global* coordinate system covering the entire spacetime is an impossibility, except in the (unphysical) case of a perfectly uniform gravitational field.

Let us summarize: In the presence of gravity one may define a local inertial frame in a sufficiently small neighborhood of any spacetime point by considering a freely falling frame. In such a frame the non-gravitational laws of Physics are the standard ones of SR (or of classical Physics in the non-relativistic limit). This reflects the fact that spacetime is locally flat, although it is curved on a larger scale due to gravity, the effects of which can only be removed locally (in a small region of spacetime where the gravitational field can be considered almost constant in space and time).

7. Concluding remarks

A (generally inhomogeneous) gravitational field adds a geometrical structure to the flat spacetime of SR; namely, it endows spacetime with *curvature*. The spacetime of GR is thus a genuine Riemannian space [8] the specific properties of which are determined by the distribution of matter and energy in space and time.

Since *global* Lorentzian coordinates with a globally constant metric $\eta_{\mu\nu}$ cannot be defined in a curved spacetime, in GR one must allow for general spacetime coordinates $\bar{x}^{\mu}$ as well as a corresponding non-constant metric $g_{\mu\nu}(\bar{x}^{\lambda})$ that cannot be reduced to $\eta_{\mu\nu}$ by any global coordinate transformation, except at any particular spacetime point (or, at best, in a small neighborhood of such a point).

Moreover, the linear Lorentz transformations of SR, relating different Lorentzian systems of coordinates that correspond to genuinely inertial frames, must be replaced by general, *nonlinear* transformations of the $\bar{x}^{\mu}$. Accordingly, the Lorentz invariance of physical laws, assumed in the gravity-free flat space of SR, must be generalized to *general invariance* under the nonlinear coordinate transformations of GR. In order for this to be achieved, the equations expressing physical laws must be re-expressed in forms that comply with the aforementioned requirement of general invariance [8].

The existence of *local inertial frames*, corresponding to local Lorentzian coordinates, is a manifestation of the *local flatness* of a curved Riemannian spacetime. In the special case where a *global* transformation from general spacetime coordinates with non-constant metric, to Lorentzian coordinates with metric $\eta_{\mu\nu}$, exists, the spacetime is flat and gravity is absent. The general coordinates then simply correspond to an *accelerated* frame of reference, relative to the inertial frames allowed by SR.

Finally, the curvature, hence the geometry, of spacetime is determined by the general metric $g_{\mu\nu}(\bar{x}^{\lambda})$ and is dependent on the distribution of matter and energy in spacetime. The relationship between this distribution and the spacetime geometry is mathematically expressed by the *Einstein field equations* (see, e.g., [3,8]), a set of differential equations for the metric components.

Appendix

A. Worldline geometry

To simplify our analysis we treat the problem within the framework of classical Newtonian mechanics and Galilean relativity. Thus our "spacetime" will actually be the Newtonian one where time is absolute (frame-independent). Our general conclusions, however, will still be valid in SR (upon necessary modifications regarding the relativity of time).

Proposition: The worldline of a *free* particle in a spacetime diagram, as well as the trajectory of this particle in space, are straight lines relative to an inertial frame of reference.

Proof: We assume for simplicity that the motion of the free particle is confined to the *xy*-plane (z=0). Relative to an inertial frame the particle moves with constant velocity. The equations of motion are thus linear in time, of the form

$$x=\alpha t+\beta\ , \quad y=\gamma t+\delta \tag{A.1}$$

with constant α, β. Relations (A.1) describe a straight worldline in a spacetime diagram. To find the equation of the trajectory we eliminate t between the two equations in (A.1). The result is

$$y=\kappa x+\lambda \quad \text{where} \quad \kappa=\gamma/\alpha\ ,\ \lambda=\delta-\beta\gamma/\alpha\ ,$$

describing a straight-line path in the *xy*-plane.

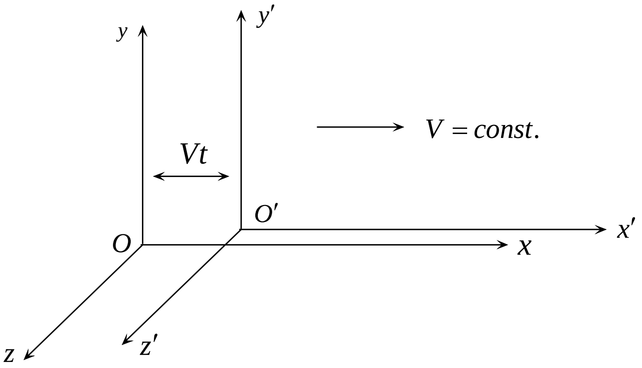

Fig. 1

Let O be the origin of the inertial frame within which the equations of motion (A.1) apply. We consider a second inertial frame with origin O', the axes of which frame are parallel to those of O (see Fig. 1). The two systems of axes coincide at time t=0, while for t >0 the frame O' is moving with constant velocity V relative to O along the common x-axis of the two frames. The coordinate transformations relating the two frames are

$$x'=x-Vt\ ,\ \ y'=y\ ,\ \ t'=t \quad \Leftrightarrow \quad x=x'+Vt'\ ,\ \ y=y'\ ,\ \ t=t' \tag{A.2}$$

To find the equations of motion of the free particle relative to the frame O', we substitute x, y and t from (A.2) into (A.1):

$$x' = (\alpha - V)\,t' + \beta\,, \qquad y' = \gamma t' + \delta\,,$$

which are linear equations in t'. By eliminating t' we can find the equation for the trajectory in the $x'y'$-plane, which is again of the form $y' = \kappa x' + \lambda$. Therefore, a straight-line path in the *O*-frame is transformed into a straight-line path in the O'-frame, both frames being inertial.

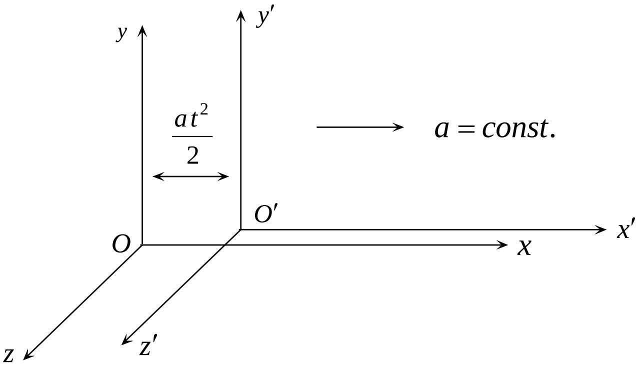

Fig. 2

We next consider the case where the frame O' is accelerating in the *x*-direction relative to the inertial frame *O*, with constant acceleration *a* (Fig. 2). For simplicity we assume that the initial velocity of O' relative to *O*, at the moment t=0 when *O* and O' coincide, is zero. The coordinate transformations relating the two frames are

$$x' = x - at^2/2\,,\quad y' = y\,,\quad t' = t \quad \Leftrightarrow \quad x = x' + at'^2/2\,,\quad y = y'\,,\quad t = t' \qquad \text{(A.3)}$$

The equations of motion of the free particle relative to the non-inertial frame O' are found by substituting *x*, *y* and *t* from (A.3) into (A.1):

$$x' = -\,at'^2/2 + \alpha t' + \beta\,, \qquad y' = \gamma t' + \delta\,.$$

By eliminating t' we can find the equation for the trajectory of the free particle in the $x'y'$-plane, which is an equation of a parabola:

$$x' = \kappa y'^2 + \lambda y' + \mu\,.$$

Conclusion: A straight path relative to an inertial frame appears *curved* in a non-inertial frame. Equivalently, a straight path in a non-inertial frame appears curved relative to an inertial frame.

The above remarks lead to an explanation of the effect of bending of light in a gravitational field.

B. Bending of light by gravity

This can be viewed in two ways.

1. View of Ellis & Williams [6]:

In an *inertial* frame *away from gravity*, light travels in straight lines. By the equivalence principle, the same must be true for a *freely falling* frame inside a gravitational field. Indeed, as mentioned previously, since light carries energy it has inertia and therefore possesses an effective “gravitational mass”; it is thus affected by gravity like any material particle. Now, as we saw in Sec. 6, any physical entity (including light) subject to no other interactions except gravity appears to travel in a straight-line path inside a freely falling lab.

On the other hand, relative to an *inertial* observer *inside* a gravitational field (i.e., someone who is *stationary* rather than free-falling), a path that seems straight in a falling lab will appear to be *curved*. Thus the stationary observer (e.g., one on the surface of the Earth) will conclude that the gravitational field has the effect of *bending* the path of light.

2. View of Susskind [9]:

Again, light travels in straight lines in an inertial frame away from gravity. However, the path of light will appear to be *curved* relative to an *accelerated* frame. On the other hand, the accelerated frame *outside* gravity is locally equivalent to an inertial frame placed *inside* a gravitational field (cf. Sec. 3). Therefore, the observer in the latter frame will conclude that the gravitational field bends the path of light.

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