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MAXWELL, EINSTEIN AND HIGGS

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The hidden symmetry and Mr. Higgs!

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Abstract. Written in non-technical language, this review article explains the significance of the Higgs field and the associated Higgs boson in High-Energy Physics. The connection of symmetry with particle interactions and their unification is also discussed in this context. The presentation is informal and physical concepts are demonstrated through metaphors from everyday experience.

1. Introduction

One of the dominant scientific issues in 2012 and 2013 was the experiments that took place at the **CERN** research center in Geneva. Their major goal was the experimental verification of the existence of a mysterious particle which constitutes a fundamental ingredient of the model that we believe describes the elementary building blocks of matter and the interactions among them. The **Higgs boson** (the quantum of the **Higgs field**) was indeed the biggest bet of the research efforts, and the verification of its existence – pending some remaining issues, to be resolved in the near future – was hailed as a triumph of High-Energy Physics and, predictably, led to long-awaited and well-deserved Nobel prizes [1].

But, why was this elusive particle so important as to justify spending several billion dollars for its hunt at a time of economic world-crisis? Well, if not for anything else, perhaps for a deep sigh of relief of physicists – at least those who didn't bet on the collapse of modern High-Energy Physics in order to be given the historic chance of building it from the start!

This article constitutes an attempt to explain, in the simplest terms possible, the reason why the Higgs field and the associated Higgs particle are such important elements of contemporary physics theories that try to unlock the mysteries of the world that surrounds us. And, given that the matter we observe, at the most fundamental level, is made of **elementary particles** [2] (such as, e.g., the familiar *electron*, as well as others "residing" in the atomic nucleus), we begin our story by examining the different ways these particles interact with one another...

2. The hidden simplicity of Nature

In accordance with the phenomenology of the low-energy world we live in, we can distinguish four kinds of **forces** (or **interactions**) [2] among the elementary constituents of matter:

(1) **Gravitational forces**, which are responsible for the weight of all objects, as well as for the motion of the Earth around the Sun.

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(2) **Electromagnetic forces** (a force of such origin is, e.g., the friction we feel when we rub our hands against one another).

(3) **Strong forces**, to which the atomic nucleus owes its coherence despite the repulsion between the positively charged protons.

(4) **Weak forces**, responsible for a number of processes taking place inside the nucleus.

There are indications, however, that Nature is much simpler than it appears to be! For example, prior to the systematic theoretical formulation of the laws of electromagnetism by **James Clerk Maxwell** (1831-1879), electricity and magnetism were treated as two distinct physical phenomena, independent of each other. This was due, in part, to the apparent differences in the properties of electric and magnetic forces.

With his complex mathematical equations [3,4], Maxwell described the electric and the magnetic field as “two sides of the same coin”, since each field may transform into the other, depending on the way we observe it. Hence, instead of two separate fields (electric and magnetic), we now speak of a single **electromagnetic field**.

It is interesting to note that (see, e.g., p. 588 of [5]), with regard to their relative strengths, the electric and the magnetic force become equivalent to each other in the limit of high speeds – thus, high energies – of the interacting electrical charges. This is a first hint that *the simplicity of Nature reveals itself to us only when sufficient energy is spent for its experimental observation!*

One of the biggest achievements of twentieth-century Physics was the discovery that, in a similar way, the electromagnetic and weak forces also represent two manifestations of a single interaction, the **electroweak force**. The problem of an even larger unification incorporating the strong interaction as well remains an open challenge. Gravity, on the other hand, is a different kind of problem since, in contrast to the other forces, it doesn’t lend itself easily to a quantum formulation (see, e.g., [6]).

From the experimental point of view, the thing to keep in mind is that, as mentioned above, the simpler Nature appears to be through these successive stages of unification, the more expensive is the “ticket” the spectator of this simplicity is required to pay. And the name of this ticket is *energy* ! That is, the supposed simplicity of Nature can only be revealed through very-high-energy experiments. And, the greater is the degree of simplicity, the more is the energy required. This explains the enormous expenditure for the construction of bigger and bigger elementary-particle accelerators, like the **Large Hadron Collider** (LHC) at CERN [7].

3. The other side of the hill

A simple example may help us better understand the situation: Imagine you reside at the foot of a hill located at the center of a town, the houses of which are exactly similar to each other and are uniformly distributed around the hill. From the point you are located, only a part of the town is visible since the hill blocks the view to the other side. Thus, from your point of view, there is “your” neighborhood and some other one, at the opposite side of the hill. At the point where you stand, your perception of the town is partial and *asymmetric*.

Suppose now you find the strength (that is, the required energy) to walk up to the top of the hill. From there you can look around and see every neighborhood of the town. The view is now complete and perfectly *symmetric* (no matter how you turn your body, you will always see some part of the town and, according to our assumption, all parts look alike). What we must keep in mind is that, *moving from the complexity of asymmetry toward the simplicity of symmetry requires the expenditure of energy!*

4. The symmetry behind the interaction

The elementary particles and the interactions (forces) among them are described by the so-called **Standard Model** [2,8]. This model is basically a synthesis of all experimentally verified theories on the structure of matter at the most fundamental level. An issue in need of experimental verification was the mechanism by which the particles (and, macroscopically, matter itself) acquire **mass** (or, if you prefer, **inertia**). Well, you may ask, isn’t mass an inherent property of each particle, endowed to the particle from the very beginning of its creation? To understand the problem, it is necessary to go back to the concept of symmetry...

In the microworld, symmetry is much more than just a matter of aesthetics! Among other things, it is the factor that determines the kind of interaction between particles. That is, *behind every form of interaction there is a corresponding symmetry*, where by “symmetry” we mean **invariance** of some sort under certain mathematical transformations (see Appendix). As an example, the electromagnetic interaction between electrically charged particles can be associated with the symmetry (invariance in form) of the fundamental equations of Electromagnetism under specific abstract mathematical transformations of the functions that describe the electromagnetic field and the particles interacting through it [2].

According to quantum theory, the electromagnetic field itself is represented by its own “particles”, **photons**. We can think of them as little spheres of energy exchanged between charged particles, making one particle aware of the presence of another. Photons are the **quanta** (the most elementary quantities) of the electromagnetic field. Their role is to communicate the electromagnetic interaction between electrically charged particles.

With regard to symmetry, the photon plays the role of a “messenger” who informs every observer by whom it passes about the details of the mathematical transformations performed on the functions representing the particles at neighboring points of space (or, more correctly, of *spacetime*).

A serious constraint, however, must be taken into account: The theories associating particle interactions with underlying symmetries demand that the quanta of the field responsible for an interaction should have **zero mass** [2]. This is indeed true for the photons (carriers of the electromagnetic interaction) but not for the quanta of the field associated with the weak interaction. Thus, the latter interaction would be at risk of staying out of the game of symmetry, and the theory of the unification of the weak force with the electromagnetic (electroweak force) would break down... if a mysterious field weren’t there to save the game!

5. The boring professor and his popular escort!

The mass problem is dealt with by introducing the **Higgs field**. This field allows us to regard the quanta of all interactions as *intrinsically* massless. Their *apparent* property known as mass is due to their interaction with the Higgs field, or, if you prefer, with the quantum of this field, the **Higgs boson** [2,9]. (The term *“boson”* refers to particles with the property that any number of them can occupy the same quantum state. This is a fundamental property of the quanta of all interactions. This is *not* the case, however, with electrons or other *matter* particles such as *neutrinos* or *quarks* [2]!) Generally speaking, as proposed by **Peter Higgs** and other theoretical physicists working independently, the mass of any elementary particle is an *acquired* property that originates from the particle’s interaction with the ubiquitous Higgs field.

Thus, hypothetically, if someone suddenly “turned off” this field (as we assume the case was for a small period after the Big Bang [6,10] due to extreme temperatures), all particles would appear *massless* (they would have no inertia, that is, they would not resist any attempt to alter their state of motion). According to the Theory of Relativity, this would mean that every particle would travel at the speed of light. We know, of course, that this isn’t true in reality (with the exception of the photon).

Again, an example will be helpful: Imagine a ball organized by university students. In the big dancing room, a large number of students are uniformly distributed all over the place. Let us suppose that this multitude of students constitutes the “Higgs field” and each individual student represents a “Higgs boson” (a quantum of the field).

At some point in the evening, a boring professor (say, the author) makes his appearance at the ball. As he gets little attention upon entering the room, he can move more or less freely and accelerate almost at will. He is a “particle” with a small mass (a small inertia) since the Higgs field and its quanta (the students) do not bother much to slow down his motion!

Imagine now the late arrival of the professor’s beautiful lady escort. As she attracts the attention of the students, they all rush to approach her, thus making it difficult for her to move inside the room. So, in order to speed up her step she will need to exert force: the Higgs field (the students) endowed her with a large mass (inertia). (Since this is only an allegoric paradigm, it should not be concluded that the lady is, literally, overweight!)

Now, if the students somehow became invisible, then an external observer might *assume* that this inertia is an inherent property of the woman. In a similar spirit, we presume that the inertia exhibited by all bodies is not an intrinsic property but simply a result of their interactions with the “invisible” (under normal, low-energy conditions) Higgs field. And this field becomes “visible” through its quantum, the Higgs boson.

6. Epilogue

So, the latest experiments appear to confirm the Higgs theory, although several issues remain open and are in need of further investigation [11]. The delay in the discovery of the Higgs boson was due to the fact that this particle is extremely heavy, as it interacts strongly with its own field! Thus, its creation in the laboratory demands very high energies (remember the famous Einstein relation according to which mass and energy are equivalent). It was to this end that the LHC was built at CERN.

Physicists can now rejoice at the happy outcome of this enormous scientific endeavor, which – on top of everything – was excessively costly at a time of international economic crisis. What would be the consequences had these experiments failed to verify one of the fundamental predictions of the Standard Model? Well, a large part of Particle Physics as we know it would probably have to be revised and new approaches would have to be considered. It must be said, however, that, for some physicists such a scenario wouldn’t necessarily be catastrophic! Any scientific theory is good for as long as it is supported by experiment. The experimental overturn of a theory, unpleasant as it may be, opens new paths and creates new opportunities in scientific research. Isn't this what happened, in an almost cataclysmic way, at the beginning of last century?

Acknowledgment

I thank my students for penetrating questions that prompted me to write this article.

Appendix

In Physics, a *field* is the assignment of a definite value to a physical quantity, for each point of spacetime. Thus, for example, the electromagnetic field is represented by a pair of vectors ($\boldsymbol{E}$, $\boldsymbol{B}$), each of which takes on a certain value at each spacetime point (x,y,z,t). The manner in which these vectors change in space and time is described by a set of *differential equations*, called *Maxwell's Equations* [3-5]. In general, every field is associated with a corresponding differential equation (or set of differential equations) such that the field (viewed as a mathematical function) is a *solution* to this equation.

A transformation of a field leaving the corresponding differential equation *invariant in form* is said to represent a *symmetry* of this equation. Thus, a symmetry transformation produces a new solution of the field equation from any given solution. The fields that represent particle interactions emerge by demanding that the field equations for the interacting particles be *invariant* under certain groups of local transformations. (To be accurate, this invariance concerns the *Lagrangian function* associated with these equations.)

For more details on symmetries of differential equations, in general, the reader is referred to [12] and the extensive references therein.

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Electromagnetic waves, gravitational waves and the prophets who predicted them

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Abstract. Using non-excessively-technical language and written in informal style, this article introduces the reader to the concepts of electromagnetic and gravitational waves and recounts the prediction of existence of these waves by Maxwell and Einstein, respectively. The issue of gravitational radiation is timely in view of the recent announcement of the detection of gravitational waves by the LIGO scientific team.

1. Introduction

Undoubtedly, *James Clerk Maxwell* (1831-1879) and *Albert Einstein* (1879-1955) were the leading figures of Theoretical Physics in the past two centuries. Among their many achievements, Maxwell unified electricity and magnetism into a singe electromagnetic theory and predicted the existence of *electromagnetic waves*, while Einstein's Relativity changed our conception of space and time and led to modern gravitational theory, in the context of which *gravitational waves* were predicted to exist.

Unfortunately, Maxwell didn't live long enough to see the experimental confirmation of existence of electromagnetic waves. As for Einstein, in a sense he was "luckier" since it would be biologically impossible for him, anyway, to be present in the official announcement of the detection of gravitational waves one whole century after he had predicted them to exist!

In this article we will try to explain the nature of electromagnetic and gravitational waves and examine how they are produced. For further and deeper study of the subject the reader is referred to the sources cited at the end.

2. Maxwell and the first unification theory for interactions

As is well known, every electric charge (regardless of its motion) produces an electric field and is itself subject to a force inside an existing electric field. Also, every *moving* charge produces a magnetic field and experiences a force inside such a field.

We often tend to view *electricity* and *magnetism* as two separate natural phenomena. Indeed, in a hypothetical world where all electric and magnetic fields remained unchanged with time, there would be no way of knowing that electric and magnetic phenomena are interrelated and mutually dependent. At the theoretical level, the set of four *Maxwell equations* [1,2] would break into two *independent* pairs, one for the electric and one for the magnetic field.

In 1831, in a series of experiments, *Michael Faraday* discovered something interesting: Whenever a magnetic field changes with time, an electric field emerges! Although there were no experimental indications at the time, Maxwell assumed that the converse is also true; that is, a magnetic field is present whenever the electric field varies with time. Thus, given this mutual dependence between the electric and the magnetic field, electric and magnetic phenomena should not be treated as separate.

Historically, this was the first *unification theory* of apparently different interactions – electric and magnetic – into a single *electromagnetic interaction*. In the twentieth century the unification scheme would be enhanced by incorporating the weak and the strong interaction – and by making a heroic, albeit frustrating, effort to include gravity as well...

3. Electromagnetic waves

With his mathematical genius, Maxwell "codified" the laws of electromagnetism with a system of four equations (expressed in differential or, equivalently, in integral form) that describe the behavior of the electromagnetic field in space and time [1,2]. One consequence of these equations is the conclusion that the electromagnetic field must exhibit wavelike properties. That is, a change ("disturbance") of the field at some point of space is not felt instantaneously at other points but propagates as an *electromagnetic wave* traveling at the speed of light. In particular, light itself is just a special type of electromagnetic wave having the additional property of being sensed by our eyes.

The importance of electromagnetic waves for our lives cannot be overemphasized! Through them we receive light and warmth (and, unfortunately, some harmful radiation as well) from the Sun, we enjoy stereo music on the radio, we watch football matches on TV, we communicate by using our cell phones... But, how are these waves produced in the first place?

First, some terminology: The propagation of energy by means of electromagnetic waves is called *electromagnetic radiation*. (Henceforth we will write *"e/m wave"* and *"e/m radiation"*, for short.) Thus, a physical system that emits energy in the form of e/m waves is said to *emit e/m radiation* or, simply, to *radiate*. Such systems include atoms, molecules, nuclei, hot bodies, radio-station antennas, etc.

By a careful examination of the Maxwell equations it follows that the e/m radiation is produced in basically two ways: (*a*) by *accelerated* electric charges and (*b*) by time-varying electric currents [1,2]. In particular, a non-accelerating charge (one that moves on a straight line with constant speed) does *not* radiate. I often explain this to my students (before writing any equations) by using the following parable:

On a hot summer day you go to the store and buy an ice cream. You decide to eat it on the road before it melts. You take a carefree walk on a straight path, with steady step (thus, with *constant velocity*), without noticing a swarm of bees following you (or, rather, your ice cream)! When you suddenly notice them, you *accelerate* your motion in order to escape from them (you either move faster in the same direction or just change your direction of motion). Scared by this move of yours, some of the bees leave the swarm and fly away, never to come back...

What is the meaning of all this? The “ice cream” is an electric charge initially moving with constant velocity and carrying with it the total energy of its e/m field (the “swarm of bees”). This is just a transfer of a constant amount of energy in the direction of motion of the charge. When the charge accelerates, a part of this energy (the “bees” that fly away) is detached, in a sense, and travels to infinity at the speed of light in the form of an e/m wave. And, the higher the acceleration of the charge, the greater the energy radiated per unit time.

4. Einstein and Relativity

In empty space, the speed of light (denoted *c*) is approximately 300,000 kilometers per second. Now, speeds (and, more generally, velocities) are determined relative to some *frame of reference*. For example, when a passenger walks along the corridor of a moving bus, her speed as measured by a seated passenger is different from that which would be recorded by someone standing on the sidewalk. The bus and the sidewalk define two different frames of reference relative to which the velocity of the walking passenger is determined.

So, relative to which frame of reference does the speed of light have the familiar value *c*? Based on perceptions at his time, Maxwell assumed that the speed of propagation of e/m radiation takes on the “correct” value *c* in a privileged frame of reference that is at rest relative to the *ether*, a hypothetical substance with almost metaphysical properties that was believed to occupy the whole of space. It is in this frame only that the Maxwell equations would assume their proper form. Thus, any observer moving relative to the ether should measure a speed of light different from *c* and should also conclude that the electromagnetic phenomena are not correctly described by Maxwell’s equations.

However, every attempt to experimentally verify the dependence of the speed of light on the state of motion of the observer failed. Then, in a historic article of 1905, Einstein proposed that the speed of light in vacuum has the *same* value *c* for *all* observers, regardless of their state of motion. Moreover, the laws of Physics – and, in particular, the Maxwell equations of electrodynamics – should assume the same form in all frames of reference. (Technically speaking, the above principles are valid for a special class of *inertial observers* associated with *inertial frames of reference*.) These principles form the basis of the *Special Theory of Relativity*.

In classical (Newtonian) mechanics, time has absolute meaning, common to all observers. Thus, according to this theory, if a pulse of light is emitted from one point of space toward another, different observers will agree with one another with regard to the time it took for light to make the journey, although they will possibly disagree on the distance traveled (each observer will measure this distance *relative to himself*).

In Relativity, however, these observers must agree with one another regarding the speed c of light. Given that they will disagree, in general, about the length of the route, they must now also disagree with regard to the time taken for light to make the trip. Thus, Relativity puts an end to the idea of absolute time. Time intervals as well as spatial distances are directly dependent on the motion of the observer and are devoid of absolute meaning.

In addition to this, the constancy of the speed of light imposes a sort of mathematical interweaving between space and time coordinates of an event, such that the distinction between space and time is also not absolute but depends on the motion of the observer. Thus, in place of the separate terms *"space"* and *"time"*, in Relativity one speaks of *spacetime*.

Special Relativity did not change the classical form of the Maxwell equations; however, it dramatically revised Newtonian Mechanics, which was now seen to be valid as an approximation in the limit of "small" speeds (in comparison, that is, to the huge value of c). Among other things, Relativity revealed a remarkable relationship between mass and energy ($E=mc^2$), which has no classical analog. As we know, an early "experimental" verification of this relation costed countless human lives in Hiroshima and Nagasaki at the end of WW2.

5. Gravity is... geometry!

The spacetime of Relativity is four-dimensional, with three dimensions corresponding to space and one to time. Let us consider, for simplicity, just two dimensions, one for space and one for time. In Special Relativity the geometry of such a two-dimensional spacetime would *look like* that of an infinite plane surface (although the mathematical recipe for evaluating distances would be somewhat different). With regard to intrinsic geometrical properties, such a *flat* surface has fundamental differences from curved surfaces such as, e.g., the surface of a sphere.

In 1915, Einstein proposed his *General Theory of Relativity*, which was based on a very original idea: What we perceive as *gravity* is not, in reality, a force (like, e.g., electric or magnetic forces) but is a manifestation of a *geometric deformation of flat space* (technically speaking, of flat *spacetime*) caused by the presence of matter [3]. Thus, for example, the observed motion of the Earth around the Sun is not – as Newton would assert – due to the gravitational force exerted on the Earth by the Sun but is due to the *curvature of space* caused by the mass of the Sun itself. Space has no longer geometrical properties similar to these of a flat surface but rather similar to those of the curved surface of a sphere.

So, in General Relativity the gravitational field is not treated as a force field but rather as a field of deformations (“ripples”) in the fabric of spacetime. And, locally, these deformations are greater the greater the mass that causes them.

6. Gravitational waves

What happens when the gravity-related ripples at some region of space change in time due to a redistribution of matter in that region? Let us recall the situation in electromagnetism: Every redistribution of the sources of the e/m field (charges and/or currents) in a region of space causes a disturbance of the e/m field in that region, which disturbance propagates in space as an e/m wave at the speed of light, *c*. In particular, an *accelerated* electric charge emits energy in the form of e/m radiation. The emitted e/m wave thus takes away a part of the charge’s total energy.

Now, as Einstein showed in 1916, a consequence of the equations of General Relativity is that any redistribution of matter in a region of space causes a disturbance of local geometry (in classical terms, of the gravitational field), which (disturbance) propagates in space as a *gravitational wave* [3–6] traveling at speed *c*. Moreover, an *accelerated* body loses part of its energy, as it becomes the source of *gravitational radiation*.

The problem is that, whereas even an atomic system may emit detectable e/m radiation (e.g., visible light), the production of detectable gravitational radiation requires enormous masses with very high accelerations. Such physical conditions do exist in the Universe (rotating pairs of neutron stars or black holes, stellar collisions and explosions, etc.) and the liberated gravitational energy is indeed huge. These phenomena, however, (fortunately!) occur so far from us that, by the time they reach the Earth, the emitted gravitational waves will be millions of times weaker. Thus, an exceptionally sensitive device is needed in order to detect these waves.

7. Why are they useful?

Until recently, all information we obtained about the Universe was based on observations via e/m radiation (visible light, radio waves, microwaves, X-rays, etc.). Gravitational waves may now provide information that would otherwise be impossible to get. For example, a collision of black holes does not produce an appreciable amount of e/m radiation while it does produce enormous gravitational radiation. Thus, with the aid of gravitational waves we will be able to study such catastrophic cosmic phenomena.

Also, in contrast to e/m radiation, which interacts strongly with matter – thus is subject to absorption and distortion as it crosses distances of millions of light-years in the Universe – gravitational waves can travel huge distances practically unchanged (they only weaken in magnitude as they spread in space while moving away from the source). Thus, the information these latter waves provide is much more faithful compared to that furnished by e/m waves.

Finally, gravitational waves are expected to provide answers to some important cosmological questions regarding the early stages of evolution of the Universe. Traditional astronomy is not in a position to answer such questions, as the Universe was initially opaque to e/m radiation and thus no information of electromagnetic origin may reach us from that cosmic period.

8. Epilog: Why all this excitement lately?

Even though Einstein had predicted the existence of gravitational waves as early as in 1916, an *indirect* astronomical confirmation of their existence was obtained much later, in the 1970s. However, there had never been a *direct* detection of such waves on Earth.

On February 11, 2016, the scientific team of *LIGO* (*Laser Interferometer Gravitational-wave Observatory*) [4] announced that they had detected gravitational waves on September 14, 2015. These waves were produced by the merging of two black holes (which initially formed a rotating pair) at a distance of *1.3 billion light-years* from Earth [7]. It was the ultimate confirmation of Einstein's General Relativity!

The LIGO "observatory" consists of two identical detectors (*laser interferometers* [4]) located in the USA, with a distance of about 3,000 kilometers between them. *LIGO Hanford* is in southeastern Washington State; *LIGO Livingston* is in Louisiana.

Certainly this is not the end of the story. It is only a small but decisive first step in man's deep cosmic experience, the beginning of an ambitious endeavor to explore the *"final frontier"* of space – *"to boldly go where no man has gone before"* !

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[7] Because of its rotation, the system of the two *accelerating* black holes continuously lost energy as it emitted gravitational waves. This had the effect of diminishing the distance between the two objects, which fact made the system spin faster and faster and emit more and more gravitational radiation in the process, which again forced the two bodies to get closer and closer, and so forth. At the final stage of the process, the two objects collided and merged, emitting a huge quantity of gravitational energy within a very short time.

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But, how can the atom be so stable, Dr. Maxwell?

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Abstract. In the initial stages of its development, atomic theory had to bypass the laws of classical electromagnetism in an *ad hoc* manner in order to explain the stability of atoms. In quantum mechanics, however, the classical theory may find again some room even for a microscopic structure such as the atom. Provided, of course, that certain classical concepts are reexamined and suitably reinterpreted...

1. Electromagnetic radiation: a triumph of classical Physics

There is no doubt that *James Clerk Maxwell* (1831-1879) was the leading figure of Theoretical Physics in the nineteenth century. Among his many achievements, Maxwell unified electricity and magnetism into a single electromagnetic theory and predicted the existence of electromagnetic waves. Unfortunately, Maxwell didn't live long enough to see the experimental confirmation of his prediction...

We often tend to think of electricity and magnetism as separate natural phenomena. And, indeed, they exhibit fundamental differences. For example, even stationary electric charges "feel" the electric interaction whereas only moving charges are subject to the magnetic interaction. In a hypothetical world where all electric and magnetic fields were static (i.e., time-independent) there would be no way of knowing that electric and magnetic phenomena are interrelated and mutually dependent. From a mathematical point of view, the famous four Maxwell's equations would split into two independent pairs corresponding to the electric and, separately, the magnetic field (see, e.g., Chap. 9 of [1]).

In 1831, however, Michael Faraday experimentally discovered something interesting: a time-change of a magnetic field is necessarily accompanied by the appearance of an electric field! Despite the lack of experimental evidence at his time, Maxwell predicted that the converse was also true; namely, a magnetic field appears each time an electric field changes with time. No absolute separation is thus possible between electric and magnetic phenomena, given that the electric and the magnetic field appear to be intimately related.

Historically speaking, this has been the first unification theory of seemingly independent interactions – the electric and the magnetic – into a single *electromagnetic* (e/m) interaction. In the twentieth century there would be a further enhancement of the unification scheme with the inclusion of the weak and the strong interaction, along with a heroic effort of incorporating gravity as well.

With his mathematical genius, Maxwell was able to describe the electromagnetic phenomena in terms of a set of equations that bear his name. The four *Maxwell's equations* [1] describe the behavior of the electromagnetic field in space and time. From these equations there follows the interesting conclusion that the electromagnetic field has wavelike properties. That is, a change (or, as we say, a *disturbance*) of the field at some point of space is not felt instantaneously at other points but propagates in the form of an *electromagnetic wave* (or e/m wave, for short) traveling at the speed of light. Light itself is a special kind of e/m wave having the property that it may be sensed by our eyes.

The propagation of energy by means of e/m waves is called *electromagnetic (e/m) radiation*. A physical system that emits energy in the form of e/m waves is said to *emit e/m radiation* (or, simply, to *radiate*). Examples of radiating systems are atoms, molecules, nuclei, hot bodies, antennas of radio and TV stations, etc.

By the Maxwell equations it follows that, in principle, the e/m radiation is produced in either of two ways: by *accelerated electric charges* (regarded as isolated quantities) or by *time-varying electric currents*. In particular, a charge moving at constant velocity (i.e., executing uniform rectilinear motion) does *not* radiate. In a previous article [2] we explained this by using a parable:

> On a hot summer day you go to the store and buy an ice cream. You decide to eat it on the road before it melts. You take a carefree walk on a straight path, with steady step (thus, with *constant velocity*), without noticing a swarm of bees following you (or, rather, your ice cream)! When you suddenly notice them, you *accelerate* your motion in order to escape from them (you either move faster in the same direction or just change your direction of motion). Scared by this move of yours, some of the bees leave the swarm and fly away, never to come back...
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> What is the meaning of all this? The “ice cream” is an electric charge initially moving with constant velocity and carrying with it the total energy of its e/m field (the “swarm of bees”). This is just a transfer of a constant amount of energy in the direction of motion of the charge. When the charge accelerates, a part of this energy (the “bees” that fly away) is detached, in a sense, and travels to infinity at the speed of light in the form of an e/m wave. And, the higher the acceleration of the charge, the greater the energy radiated per unit time.

One might now ask the following question: As everyone knows, acceleration is always defined relative to some observer. If a charge accelerates relative to a “stationary” observer, this observer will see the charge emitting e/m radiation. However, relative to an observer moving with the charge, this charge is stationary (thus non-accelerating). How should the moving observer interpret the emitted radiation?

At this point we must recall the notion of an *inertial frame of reference* [3]. This is a system of coordinates (or axes) relative to which a free particle – i.e., a particle subject to no forces – either moves with constant velocity (executes uniform rectilinear motion) or otherwise is at rest. An observer using such a frame of reference is said to be an *inertial observer*. In accordance with the law of inertia (Newton’s first law) an inertial observer moves with constant velocity (does not accelerate) relative to any other inertial observer.

What makes inertial frames really special is the fact that it is only in such frames that Newton’s laws, as well as the laws of electromagnetism, are valid. In particular, an electric charge emits e/m radiation only when it accelerates relative to an *inertial* observer. An observer moving with this charge, however, is *not* inertial. Therefore, although relative to that observer the charge seems to be at rest (hence non-accelerating) the observer must still not attempt to interpret electromagnetic phenomena according to the Maxwell equations, since this would lead to the erroneous conclusion that even a charge at rest may emit radiation! In reality, of course, the charge radiates because it accelerates *with respect to the inertial observer*.

It is interesting that special relativity provides a simple proof that a charge moving with constant velocity relative to an inertial observer does not radiate. Here is this proof:

Consider a charge *q* moving with constant velocity relative to an inertial observer *O*. Consider also an observer *O´* who is moving with the charge. This latter observer is also inertial since she moves with constant velocity relative to *O*. Because *q* is at rest relative to *O´*, that observer will record just a static electric field and no e/m radiation from *q*. (We remark that e/m radiation requires a *time-varying* e/m field; see, e.g., Chap. 10 of [1].)

Let us now make the assumption that the “stationary” observer *O*, relative to whom the charge *q* moves with constant velocity, sees *q* emitting radiation. According to the principle of relativity, e/m radiation propagates with the same speed *c* (the speed of light) in all inertial frames of reference. Thus, if the observer *O* records radiation propagating with speed *c*, then the observer *O´* must also record radiation propagating with the same speed. But, as we said before, the observer *O´* does not see any radiation whatsoever! The reason for arriving at a wrong conclusion is our initial assumption that the observer *O* sees *q* emitting radiation. We thus conclude that *q* cannot emit if it moves with constant velocity with respect to the inertial observer *O*.

We note that the above line of reasoning is no longer valid if *q* accelerates relative to *O*, since the observer *O´* who moves with the charge is now not inertial and the principle of relativity cannot be used to correlate the observations of *O* and *O´*.

2. Classical Physics and atomic theory: a problematic relationship

An atomic system consists of a number of positively and negatively charged particles (the nucleus and the electrons, respectively) held together by electric forces in a manner that the system be stable (in the sense that it retains its identity) over a long period of time.

According to a theorem by Earnshaw, a system of charged particles cannot be in a state of stable static equilibrium under the sole action of electrostatic forces. The particles must therefore be in motion and, since this motion necessarily takes place within a very limited space, the direction of their velocity must be constantly changing. In other words, the particles must have at least a centripetal acceleration.

Now, here is the problem: According to classical electromagnetism, every accelerating charge emits e/m radiation, constantly losing energy in the process. Thus the classical theory predicts that, within a very short time interval the system must shrink and eventually collapse, losing its identity. Fortunately this never happens in reality, as the atomic systems are stable!

Another effect the classical theory is not able to explain is that the atomic systems emit and absorb e/m radiation in a selective manner. That is, each system absorbs and emits very specific frequencies of radiation. As we say, the absorption and emission spectra of the system are *line spectra*.

Where the classical theory fails, the quantum theory takes over. Let us see how this happens, taking as an example the simplest atomic system: the hydrogen atom. As a preliminary step, let us explain once more why such a system cannot be studied in the context of classical Physics.

3. Rutherford's model of the atom: an important beginning with incorrect conclusions

The first modern model of the atom was proposed in 1911 by Ernest Rutherford. In the simplest case of the hydrogen atom the sole electron revolves about the nucleus (proton) in a circular orbit of arbitrary radius, having constant angular velocity.

The picture is reminiscent of the motion of a planet around the Sun, or the motion of a satellite around a planet. There is, however, a basic difference. In the case of the hydrogen atom the motion is governed by an e/m interaction (the Coulomb force between electron and proton), not by gravity. And, in view of the electron's centripetal acceleration the classical theory predicts that the atom must constantly emit e/m radiation. As a result of this loss of energy the radius of the electronic orbit must decrease continuously (cf. Chap. 1 of [1]) until finally the electron will fall into the nucleus and the atomic structure will collapse in about 10^{-8} seconds! This, of course, does not agree with the physical observation that the hydrogen atom is stable.

But, this is not the end of the story. During a continuous change of the size and the energy of the atom, the frequency of the emitted radiation must also change in a continuous manner [1]. As mentioned previously, however, the atoms do not emit e/m radiation within a continuous spectrum of frequencies but, instead, each atom emits a specific set of frequencies that constitutes a hallmark of the atom. In other words, the emission spectra of atoms (and likewise of molecules) are *line spectra*.

So, although an important first step toward understanding atomic structure, Rutherford's model can explain neither the stability of the atom nor the non-continuity of the atomic spectra. And here comes quantum theory – with its own initial problems...

4. The Bohr model: an amalgam of classical and quantum ideas

In 1913 Niels Bohr presented a modification of the Rutherford model for the hydrogen atom by proposing a new model that combined classical concepts, such as the trajectory of a particle, with novel ideas like the quantization of angular momentum and energy.

In a rather *ad hoc* manner, Bohr enhanced the Rutherford model by adding two quantum rules:

1. The electron is not allowed to follow arbitrary circular paths around the nucleus but, instead, it must describe orbits of well-defined radii. Along these orbits the electron does *not* emit e/m radiation and the energy of the hydrogen atom assumes specific, constant values.

2. The atom emits radiation only when the electron falls from an orbit of higher energy to a smaller orbit of lesser energy. The energy is emitted in the form of a single *photon* (a quantum of e/m radiation).

Bohr's theory was able to explain the line spectrum of hydrogen, giving correct values for the observed frequencies of the emitted radiation. The line property of the spectrum can be understood in the following way: In a transition of the electron from an orbit of energy E to an orbit of lesser energy E' the atom emits a photon of frequency $v=(E-E')/h$, where h is Planck's constant. And, since E and E' assume *discrete* rather than arbitrary values (that is, the energy of the atom is *quantized*), the same must be true with regard to the frequencies v of the emitted e/m radiation. We thus conclude that the line property of the emission (and likewise the absorption) spectrum is a direct consequence of the quantization of energy.

Bohr's model is not free from problems. Here are two major ones:

1. While it correctly explains the emission spectrum of the hydrogen atom, it cannot do the same thing for atoms having two or more electrons.
2. It does not answer the question of why the electron does not emit radiation when moving on the Bohr orbits despite its having centripetal acceleration.

Both these issues are treated successfully by quantum mechanics. It is the second, stability issue on which we will concentrate.

5. How quantum mechanics explains the stability of Bohr's atom

According to classical electromagnetism, a point charge in uniform circular motion emits radiation because of its centripetal acceleration. On the contrary, a constant circular current does *not* radiate since the e/m field it produces is only a static magnetic field. As mentioned earlier, the existence of e/m radiation requires that the underlying e/m field be time-dependent (sources of static fields do not radiate).

In quantum mechanics, however, the picture of a point charge moving in a definite way on a well-defined orbit is meaningless since, by the *uncertainty principle*, it is not possible to know the exact position and velocity of an elementary particle. Instead of classical orbits, quantum mechanics speaks of *stationary states* of well-defined energies. And, the motion of an electron on a definite path around the nucleus is replaced by a *probability current* related to the possible positions the electron may occupy. When the electron is in a stationary state, the corresponding probability current is constant in time.

Moreover – and this is a crucial step – the probability current may be considered as proportional to an actual electric current around the nucleus.[1] In a stationary state this latter current is constant in time. And, classically, a constant current cannot be the source of e/m radiation.

Let us specify to the hydrogen atom. The allowed Bohr orbits, on each of which the electron has a well-defined energy, correspond to the stationary states of quantum mechanics. In these states the electronic motion assumed by Bohr is equivalent to a constant electric current. Hence, in the "Bohr states" the atom does not radiate unless the electron makes a transition from a state

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[1] The advanced student may look into Chap. 3 of [4] (an old but classic textbook).

of higher energy to a state of lower energy, in which case the atom will emit a photon of frequency proportional to the difference in energy between the two states.

Now, when the hydrogen atom is not subject to external excitation, its electron “prefers” to be in the state of lowest energy, corresponding to the first Bohr orbit (the one closest to the nucleus). And, since no further transitions to states of lower energies are possible, the electron remains in the ground state and the atom no longer radiates. The energy of the atom stays fixed and the system avoids a catastrophic collapse. Stability is thus guaranteed.

So, by associating the semi-classical Bohr orbits with the stationary states of quantum mechanics, and by considering the quantum probability current as equivalent to an actual electric current, we are able to reconcile Bohr’s theory with quantum mechanics and to explain, in essentially classical terms, why Bohr’s atom is a stable system. The underlying idea is simple:

$$\text{Stationary state} \Leftrightarrow \text{stationary current} \Leftrightarrow \text{no radiation} \Leftrightarrow \text{stability.}$$

Even Maxwell wouldn’t disagree!

6. In summary...

In the initial stages of its development, atomic theory found itself in an awkward position trying to explain the stability of the atomic structure. The reason was that the modern picture of the atom seemed to violate the laws of classical electromagnetism, which dictate that every accelerating electric charge (here, the electron) must emit e/m radiation. And, if that were to happen in reality, the atom should collapse in almost no time! Atoms, however, are known to be stable structures.

The issue of stability was finally resolved by quantum mechanics, but at a price. Standard classical concepts such as the well-defined orbit of a particle had to be abandoned due to the uncertainty principle. Or, let us better say they had to be *reinterpreted*. Thus, the semi-classical orbits proposed by Bohr for the hydrogen atom were viewed as stationary states in the context of quantum mechanics.

And here comes a miracle: As a result of this conceptual redefinition, atomic stability may be “explained” in essentially classical terms in a way that is much more accessible to the non-specialist, compared to a full-blown quantum mechanical treatment of the problem. In simple words, stationary states are equivalent to time-independent currents. And, according to Maxwell’s theory, such currents are not sources of radiation. The atom will rest comfortably in the ground state and will not collapse for lack of energy.

So, even in its classical (non-quantized) form, Maxwell’s electromagnetism is essential for understanding the “logic” of quantum systems. Given that this theory also plays a fundamental role in relativity, one may justly regard J. C. Maxwell as probably the greatest theoretical physicist before Einstein! Notwithstanding the undisputed genius of Newton, I may add...

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