Costas J. Papachristou

Department of Physical Sciences

Naval Academy of Greece

Questions & Answers in

COSMOLOGY

Piraeus, 2013

Questions & Answers in Cosmology

Q.1 What would be the fate of the solar system if the (classical) law of gravity were other than an inverse-square law?

Ans. The orbits of the planets would not be elliptical: they would either spiral in to the Sun, or escape from the Sun. This is due to the fact that only inverse-square and elastic central forces give rise to closed orbits (see, e.g., p. 93 of Ref. 2).

Q.2 In Relativity, time is not completely separate from, and independent of, space, but is combined with it to form *spacetime*. In this 4-dimensional space there is no real distinction between space and time coordinates. Explain why.

Ans. In Newtonian mechanics, if a pulse of light is sent from one place to another, different observers would agree on the time that the journey took (since time is absolute) but will not always agree on how far the light travelled (since space is not absolute). In Relativity, on the other hand, all observers must agree on how fast light travels. They still, however, do not agree on the distance the light has travelled, so they must now also disagree over the time it has taken. Thus, Relativity puts an end to the idea of absolute time. Moreover, space and time are interrelated through the invariance of the speed of light: the transformation of space and time coordinates from one inertial reference frame to another must be such that a light ray would proceed with the correct speed c in both these frames. This requirement dictates a specific law of transformation of spacetime coordinates in which space and time coordinates are convoluted. Thus, there is no absolute, frame-independent distinction between space and time coordinates.

Q.3 In Q.2 we said that, according to Newtonian mechanics, two observers may disagree on the distance an object has travelled within a certain interval of time. But, isn't length (thus also distance) an invariant quantity according to classical mechanics?

Ans. Not in general! The length of a linear object is indeed invariant (frame-independent), but to determine this quantity one has to record the coordinates of both ends of the object *at the same time*. This is not so when one measures the distance an object has travelled *within a nonzero time interval*, since, in this case, the initial and final coordinates of the object were not recorded simultaneously. This explains why velocity is not a frame-independent quantity even in the context of classical mechanics.

Q.4 By what methods can we measure the distances of stars from us?

Ans. As the Earth goes round the Sun, stars that are relatively near to us are seen from different positions against the background of more distant stars, the positions of which appear fixed. This enables us to measure directly the distance of the former stars from us: the nearer they are, the more they appear to move.

Indirect methods, developed by Hubble, are used to determine the distances to galaxies which are so far away that, unlike nearby stars, their positions appear fixed. Such distances are estimated by comparing the apparent brightness of certain types of distant stars to the brightness of nearby stars of the same kind, whose distances have been measured by direct methods.

Q.5 (a) How can we determine a star's temperature? (b) How can we determine which chemical elements are present in a star's atmosphere?

Ans. (a) From the spectrum of the star's light, and by using Wien's law. (b) By observing which colors are missing from the star's emission spectrum (presumably they are absorbed by elements contained in the star's atmosphere).

Q.6 How did Hubble discover his law of expansion of the Universe? Give a simple mathematical proof of this law.

Ans. By looking at the spectra of stars in other galaxies, astronomers found that there were the same characteristic sets of missing colors as for stars in our galaxy but they were all shifted toward the red end of the spectrum. This is attributed to the Doppler effect and suggests that the other galaxies are moving away from us. Moreover, as Hubble found, the magnitude of a galaxy's red shift is not random but is directly proportional to the galaxy's distance from us. That is, the farther a galaxy is, the faster it is moving away.

By Friedmann's *Cosmological Principle*, the Universe is homogeneous and isotropic. Thus, Hubble's law must be valid for any observer in any galaxy in the Universe. (Note that individual galaxies themselves are *not* expanding structures: it is the distance between *different* galaxies that increases.) Now, let O be any observer in the Universe, and let R be the distance of any galaxy from O. Let dR be the change of R in a time interval dt. If we assume that the expansion of the Universe is uniform (i.e., all distances increase proportionally), then, according to O, the ratio dR/R must be the same for all galaxies, and so must be $(1/R)\, dR/dt$. Thus, $(1/R)\, dR/dt=const.\equiv H$, so that $v=dR/dt=HR$ (*Hubble's law*).

Q.7 If Hubble's law is correct, then the rate of expansion of the Universe must be ever increasing, since the receding speed v between any two galaxies must be growing constantly with their separation distance R. This looks unphysical since it implies that gravity will accelerate the expansion rather than halt it, contrary to what one would expect from an attractive force. Is there something wrong with the law?

Ans. This would be a real concern if the Hubble "constant" H were constant in time! This is not so, however. Thus, $H=H(t)$. The "constancy" of H means that the ratio dR/R has the same value for all galaxies around any observer O, i.e., is independent of R, in accordance with the hypothesis that the Universe is expanding uniformly about any point. Now, by differentiating Hubble's law with respect to time,

$$\frac{dv}{dt}=\frac{d^2R}{dt^2}=\frac{dH}{dt}R+H\frac{dR}{dt}=\frac{dH}{dt}R+H^2R \;\Rightarrow\; \frac{dH}{dt}=-H^2+\frac{1}{R}\frac{d^2R}{dt^2}$$

Since the expansion must be decelerating, we must have $d^2R/dt^2<0$, so we expect that $dH/dt<0$. (If we assumed that H were constant in time, then d^2R/dt^2 would have to be positive, which would mean an ever increasing rate of expansion!)

Q.8 Explain the effect of *Cosmic Background Radiation* (*CBR*).

Ans. It has been found (Penzias & Wilson, 1961) that the Universe is filled with a low-intensity cosmic radiation that is essentially isotropic. Its spectral distribution resembles blackbody radiation that corresponds to a temperature of about 2.7 K. This radiation pervades the whole Universe and is the remnant of the E/M radiation that existed shortly after the Big Bang, when the Universe was extremely hot. Due to the expansion of the Universe and the accompanying cooling, the original E/M radiation has shifted toward the red, or longer, wavelengths, corresponding to lower temperatures. At this time, it is predominantly a microwave radiation, hence it is often called the "cosmic microwave (background) radiation".

To understand the effect, imagine that the Universe is filled with standing CBR waves with a fixed number of wavelengths. As the Universe expands, there is an increase of cosmic length scale with a corresponding increase of CBR wavelength. By Wien's law, this amounts to a decrease of the average temperature of the Universe. That is, as the Universe expands, it is constantly cooling and the CBR radiation is shifting toward the microwave region of the spectrum.

It should be noted that the expansion of the Universe is not like the adiabatic free expansion of an ideal gas, which, by the 1st law of Thermodynamics, is isothermal. The latter case of expansion assumes no interactions among the constituents of the physical system (ideal gas), which is not the case with the Universe where there is always a gravitational interaction of any body with any other.

Q.9 State the *Cosmological Principle*.

Ans. It is postulated that, apart from local irregularities, the large-scale features of the Universe are spatially homogeneous (same at all points in space) and isotropic about every point (same in all directions about this point). This is clearly not true locally but may be a good approximation on a very large scale. One particular corollary is that, matter in the Universe is distributed uniformly and isotropically. This means that the average density of the Universe is expected to be the same in all local measurements within the Universe.

If this principle is valid, we can reach conclusions about the Universe as a whole from measurements of a small portion of it which is close enough to study in detail.

Q.10 Describe the three models for the possible long-term development of the Universe. (See also Q.19)

Ans. (a) *Closed Universe*: The Universe is expanding sufficiently slowly to allow the gravitational attraction between galaxies to slow down the expansion and eventually halt it. The galaxies then start to move toward each other and the Universe contracts back to a point ("Big Crunch").

(b) *Open Universe*: The Universe is expanding so rapidly that the gravitational attraction can only slow it down but never stop it. The Universe keeps expanding forever.

(c) *Flat Universe*: The gravitational attraction exactly balances the motion of the galaxies and the Universe is expanding only just fast enough to avoid recollapse.

In the closed model, where the Universe expands and recollapses, space is bent in on itself, like the surface of the Earth. It is therefore *finite* in extent but has no boundary. In the open model, where the Universe expands forever, space is bent the other way, like the surface of saddle. So in this case space is *infinite*. In the flat model, with just the critical rate of expansion, space is flat (thus also infinite).

Note: A *closed space* has the topology of a hypersphere (of *finite extent* and with *no boundary*). An *open space* is either *finite with boundary*, or *infinite with no boundary*.

The problem is analogous to that of the firing of a rocket upward from the surface of the Earth. If the rocket has a fairly low speed, gravity will eventually stop it and pull it back toward the ground. If, however, the rocket has more than a certain critical speed ("escape velocity"), gravity will not be strong enough to pull it back, so it will keep going away from the Earth forever. This critical speed is determined by the mass and radius of the Earth. In the case of the Universe, we need to know its present rate of expansion and its present average density. If the density is less than, or at most equal to, a certain critical value, determined by the rate of expansion, the gravitational attraction will be too weak to halt the expansion. If the density is greater than the critical value, gravity will eventually stop the expansion and the Universe will recollapse.

We can determine the present rate of expansion by measuring the velocities at which other galaxies are moving away from us. This is accomplished by using the Doppler effect. On the other hand, the density due to the visible matter in the Universe is far less than the critical value. The Universe, however, must contain a large amount of *dark matter* that we cannot see directly but which we know that exists because of its gravitational influence on the orbits of stars in the galaxies. The present evidence suggests that this dark matter is not sufficient to raise the density of the Universe to its critical value.

Q.11 Quantum gravity is the combination of General Relativity and Quantum Mechanics. Now, G.R. deals with the large-scale behavior of gravitating bodies in the solar system and beyond, while Q.M. deals with the behavior of matter on a very small scale. Why then is there any need to try to relate these theories?

Ans. For basically two reasons: (a) There are physical situations, a proper understanding of which requires both theories. For example, just after the Big Bang the gravitational fields were extremely strong and the distances minute, so that both relativistic and quantum effects would have been very important. According to G.R., the Big Bang itself is a *singularity* because the density of matter was infinite. G.R. specifically excludes singularities from its domain, so a new theory, which takes quantum effects into account, is needed in order to study the Big Bang as well as other singularities, such as *black holes*. (For example, as S. Hawking has shown, black holes are not really so black when Q.M. is taken into account: particles can be radiated from these objects while, classically, these objects absorb everything and emit nothing; see Q.15.)

(b) A major problem of modern physics is the *unification* of all known forces of nature under a single master theory. The objective is to incorporate gravity into Grand Unified Theory (GUT) which attempts to unify the remaining three interactions (strong, electromagnetic and weak). But, GUT is a quantum theory. Thus, if gravity is to be included in the scheme, it must first be formulated as a quantum theory.

Q.12 Describe the stages of evolution of a star.

Ans. It has been found that hydrogen is the most abundant element in the Universe, followed by helium. At the very early stages of the Universe (about 10^{10} years ago), some of the very large number of protons (hydrogen nuclei), together with the much smaller number of deuterium, tritium, helium, and neutrons, condensed into large bodies such as stars. In the process of condensation there is a transformation of gravitational potential energy into kinetic energy, resulting in an increase of temperature (to about $10^7 K$). At such temperatures, fusion of hydrogen into helium takes place by a series of fusion reactions (*proton-proton cycle*; see Q.24). Small quantities of nuclei with higher mass, such as Li, may also be formed.

Since helium is more massive than hydrogen, the helium nuclei produced in the fusion process are carried to the center (or core) of the star by gravitational attraction, gaining kinetic energy in the process. This causes a further increase in the core temperature (up to $10^8 K$) which allows for the production of Be by helium fusion reaction. When the helium concentration is large enough, C may also be formed. As the amount of C increases, a new fusion process called the *carbon cycle* becomes important.

By the process of hydrogen and helium capture, the production of successively more massive nuclei such as O, Ne and others is possible. With the production of heavier nuclei, a further gravitational contraction of the star takes place, with a corresponding increase in the kinetic energy of the nuclei and a core temperature that approaches $10^9 K$. Under such conditions of very high density and extreme temperature, other nuclear reactions are possible that may produce nuclei of higher mass number, up to the iron group (about A=60) but not heavier, since it would no longer be energetically favorable for a fusion reaction to occur. On the other hand, some reactions produce neutrons and the latter are used for the production of nuclei beyond the iron group. This is accomplished by neutron capture followed by beta decay, by means of which some neutrons transform into protons. As time passes, the number of free neutrons available for this process decreases and the production of heavier elements becomes increasingly more difficult.

Stars in the Universe do not all follow the same sequence of events at the same rate, so that stars are presently in different stages of evolution. Stars where hydrogen burning is the dominant process at this time are called *main sequence stars*. These stars (which include the Sun) are in their first stage of evolution. Those stars in which, at present, the most important process is helium burning are called *red giants* because of their color. Stars that have evolved in the way described above are called *first generation stars*.

Instabilities that arise during the evolution and ageing of a star may result in the ejection of some of its material into interstellar space. This is what happens in a *supernova explosion* (see Q.21). The ejected material mixes with uncondensed hydrogen and other particles in space, which results in the formation of *second*- (and later) *generation stars*. Such a star is the Sun.

The heat released by nuclear reactions has the effect of increasing the pressure in the interior of the star. Stars will remain stable for as long as the heat from the nuclear reactions balances the gravitational attraction. When a star runs out of fuel, it starts to cool off and so to contract. Its further evolution is then governed primarily by gravitational forces among its components and depends critically on the star's mass. This mass is expressed in terms of a characteristic quantity, called the *Chandrasekhar mass* M_C, which is about 1.4 times the mass of the Sun. Stars with masses less than 10 M_C eventually contract to a radius of a few thousand kilometers and their density is such that electrons and nuclei are packed as closely as allowed by the exclusion principle. These stars are called *white dwarfs*. (The Sun is expected to become a white dwarf in a few billion years.) Stars with masses close to 10 M_C contract even further, crushing electrons into protons so that the latter are transformed into neutrons by the process of electron capture. Such stars are called *neutron stars* and their collapse is limited by the exclusion principle as it applies to neutrons. Neutron stars have a radius of a few kilometers. Finally, stars whose mass is larger than 10 M_C contract even further, acquiring extremely high densities and becoming *black holes*.

Q.13 How could we hope to detect black holes, as by their very definition they do not emit any light (or, generally, any E/M radiation)?

Ans. (a) A black hole still exerts a gravitational force on nearby objects. Astronomers have observed many binary systems in which two stars orbit around each other, attracted toward each other by gravity. They also observe systems in which there is only one visible star that is orbiting around some unseen companion. Some of these systems are also strong sources of X-rays. The best explanation for this phenomenon is that matter has been blown off the surface of the visible star. As it falls toward the unseen companion, it develops a spiral motion (like water running out of a bath) and it gets very hot, emitting X-rays. For this mechanism to work, the unseen object has to be very small, like a white dwarf, neutron star, or black hole. From the observed orbit of the visible star, one can determine the lowest possible mass of the unseen object. If this mass exceeds the Chandrasekhar limit, the object must be a black hole.

(b) The mass of the visible stars in our galaxy is insufficient to account for the rate at which our galaxy rotates. The observed rate of rotation must be due to the extra gravitational attraction of a large number of black holes in the galaxy.

Q.14 There is some talk about "miniature" black holes, with masses much less than that of the Sun. Such black holes could not be formed by gravitational collapse, since their masses are below the Chandrasekhar limit. How could then such objects exist?

Ans. Low-mass black holes could form only if matter were compressed to enormous densities by very large external pressures. Such conditions could occur, for example, during the explosion of a very big hydrogen (fusion) bomb. Another possibility is that such low-mass black holes might have been formed in the high temperatures and pressures of the very early Universe because of the assumed lack of perfect uniformity of the latter.

Q.15 According to S. Hawking, a black hole is capable of emitting particles and radiation as if it were a hot body with a temperature that depends on the black hole's mass. How is this possible, given that nothing can escape from within the horizon of a black hole?

Ans. The particles do not come from within the black hole but from the "empty" space *just outside* the black hole's horizon. This purely quantum mechanical effect can be explained as follows: According to Quantum Field Theory, space is filled with vacuum fluctuations of the E/M field, in which pairs of photons with energies $+E$ and $-E$ are created and then recombine within a time Δt given by the time-energy uncertainty principle. Normally, a photon with negative energy could not propagate in ordinary space, but if a vacuum fluctuation takes place near the horizon of a black hole, the position of the horizon itself being influenced by the uncertainty principle, then there is a small chance that within time Δt the negative-energy photon will end up inside the horizon, where it *can* propagate freely (the gravitational field inside a black hole is so strong that even a real particle can have negative energy there). The positive-energy photon can then escape to infinity, producing radiation that *appears* to have been emitted from the black hole. This mechanism works not just for photons but for other types of particles as well. Note however that, for matter particles, the pair of particles with energies $+E$ and $-E$ must be a particle-antiparticle pair.

Q.16 How did we come to the conclusion that the *Big Bang* ever occurred?

Ans. It has been observed that remote galaxies are moving away from our galaxy with velocities that are proportional to their distances from us. By the cosmological principle, this must be true for all observers in the Universe, regardless of location. This suggests that the Universe is uniformly expanding. (Note that individual galaxies themselves are *not* expanding structures: it is the distance between *different* galaxies that increases.)

By extrapolating backward in time, we can imagine all the galaxies coming together, until some time in the distant past when all the matter in the Universe was crowded to an extreme density. This is a condition that marks the beginning of the Universe as we know it. At that moment, the Universe suddenly began its expansion with a phenomenon called the "Big Bang". Viewed as an event, the Big Bang is a *singularity* in spacetime, since at this instant the density of the Universe and the curvature of spacetime were infinite, implying a total loss of predictability and a breakdown of all physical theories, including General Relativity itself. It may be said that this singularity was *the beginning of space and time* as we know them.

The rate of expansion of the Universe (i.e., the rate at which galaxies are receding from each other) is given by *Hubble's law*: At any time, the rate of separation (i.e., the relative speed) of any two galaxies is proportional to their separation distance at that time. By using this law, it has been estimated that the present age of the Universe (i.e., the time elapsed after the Big Bang) is about 1.4×10^{10} (14 billion) years.

Q.17 As mentioned in Q.12, the ultimate fate of a star depends critically on its mass. By what means are we able to determine stellar masses?

Ans. The most effective way to measure stellar masses is by studying the motions of stars in *binary systems* (two stars orbiting about each other) and by using Kepler's Laws. The process simplifies considerably in the case of a star-planet system, where the mass of the orbiting planet is so much smaller than that of the star that the total mass of the system is almost equal to the mass of the star. By measuring the period and size of the orbit of the planet, the mass of the star may be determined. It is by this process that the mass of the Sun is found.

Q.18 Discuss briefly the large-scale structure of the Universe. What is the role of gravitation in the formation of this structure?

Ans. From the point of view of an observer on the Earth, this structure can be studied at various levels of increasing magnitude:

1. Our *planetary system*, where planets revolve around the Sun in a region about 10 light hours across. The dynamics of the system is explained by Newton's law of gravitation.

2. A large number of bright objects or *stars*, located at different distances from the Sun. The closest are the set of three stars called *Alpha Centauri*, at a distance of 4.3 Lyr (light years).

3. The Sun forms part of a rotating conglomerate composed of about 10^{11} stars held together by their gravitational attraction and called the *Milky Way* or our *galaxy*. This is a spiral galaxy of radius of about 5×10^4 Lyr. Its stars are separated by an average distance of 10 Lyr. The Sun is located in one of the arms of the spiral, about 3.1×10^4 Lyr from the center, moving under the consolidated gravitational action of the huge number of stars in the galaxy. Its orbital velocity about the center is approx. 250 km/s.

4. The Milky Way is surrounded by several "satellite" mini-galaxies, each containing about 10^5 to 10^6 stars, called *globular clusters*.

5. Our galaxy is just one of the more than 10^{10} galaxies that have been observed. Galaxies appear grouped in *clusters* of a few tens up to a few hundred. Our galaxy is part of the *Local Group*, which is a cluster of some 20 galaxies loosely bound by gravitational forces. The closest neighbors in the Local Group are two small galaxies (about 10^{10} stars each) known as the *Magellanic Clouds*. They are about 1.4×10^5 Lyr from the Sun. *Andromeda* is one galaxy in the Local Group that is very similar to ours. It contains about 3×10^{11} stars and is about 2×10^6 Lyr distant.

6. Clusters seem to group in chain-like structures called *superclusters*, containing several thousands of galaxies in a region of the order of 10^9 Lyr in diameter. The Local Group is part of the *Local Supercluster*. The largest cluster in this structure is *Virgo*. A relatively recently (1989) discovered supercluster is the *Great Wall*.

7. The superclusters are separated by regions called *voids* where the density of matter is less than 20% of the average. Some voids are as large as 10^8 Lyr in diameter.

The question now is how these large structures formed after the Big Bang, which occurred about 1.4×10^{10} years ago, as well as which forces played a role in their formation. The weak and strong forces, being of very short range, were important in shaping the events during the first few minutes after the Big Bang. They are still responsible for nuclear processes that occur in stars. But, to explain the formation of large structures in the Universe, long-range forces are needed. About 3×10^5 years after the Big Bang, gravitation, the weakest of all forces but always attractive, became the dominant factor at large scales in the Universe and it was this that produced the large structures described above. Now, by the cosmological principle, matter in the

Universe is distributed uniformly and isotropically. The problem is then how, out of an initially homogeneous Universe, did matter begin to clump into certain structures. One possible answer is that, although cosmic matter on the average is distributed uniformly, the distribution experiences local fluctuations. These fluctuations act as "seeds" for the concentration of cosmic matter, leading to the formation of these structures.

Q.19 In the presence of gravity, 4-dimensional *spacetime* is curved. How about 3-dimensional *space*? How is the spatial curvature associated with the evolution of the Universe? (See also Q.10)

Ans. First of all, it must be made clear that by "space" we mean the totality of points in the Universe. It makes no sense to talk of space *outside* of the Universe! Similarly, spacetime consists of all events in the Universe. Thus, the Universe itself *defines* the concepts of space and spacetime. Now, 3-space may be regarded as a *space-section* of spacetime, i.e., a "surface" of simultaneity. Several possibilities exist:

a. The space-sections are *flat* (i.e., they are surfaces of zero curvature). These space-sections continue indefinitely; thus this is a *spatially infinite universe*. It will therefore contain an *infinite* number of galaxies because of the spatial homogeneity of the distribution of matter in it (cosmological principle). We stress that it is *3-space* that is flat; the full *spacetime* is *curved*!

b. Three-space is a *hyperbolic* space of constant negative curvature, $k<0$. Again, the space-sections continue indefinitely. This is also a spatially infinite universe (*open universe*) containing an *infinite* number of galaxies.

c. Three-space is an *elliptic* space of constant positive curvature, $k>0$. The space sections are then necessarily *finite*, as is the total volume of the 3-space. The Universe is *spatially closed* and contains a *finite* number of galaxies. To understand the geometry of the situation, consider the lower-dimensional example of a 2-sphere S^2. Geodesics leaving in opposite directions from a point O (say, the north pole) meet again at P (south pole), the point antipodal to O. Or, starting from O and moving along a geodesic (great circle), we arrive back at O from the opposite direction. Physically, an elliptic 3-space would mean that there may exist galaxies whose light reaches us from two completely different or even opposite directions. Or, light emitted by an observer in some direction may return to him after some time from the opposite direction.

The future evolution of the Universe depends critically on the *spatial* curvature of spacetime. If $k<0$ (hyperbolic case), the Universe is a low-density universe that *expands forever*. If $k=0$ (flat case), it is a higher-density universe that just manages to keep expanding. If $k>0$ (elliptic case), it is a high-density universe that *expands* and then *contracts* again, recollapsing into another spacetime singularity ("Big Crunch"). In the cases $k=0$ and $k<0$, the spatial sections are infinite, without edge, and the expansion is simply a continual increase of distance between every pair of galaxies in the Universe. In the case $k>0$, the spatial sections are finite but again without edge (like S^2), and the expansion (contraction) is again an increase (decrease) of distance between every pair of galaxies. We note the following:

1. The expansion takes place isotropically and without a center: every galaxy "sees" every other galaxy to be receding from it equally in all directions. Thus, we can equally well choose any galaxy to lie at the origin of our coordinate system.

2. The expansion is an expansion of the Universe as a whole; it is not an expansion *into* anything! Indeed, there is nothing outside the Universe for it to expand into, since it is the totality of all that exists. Thus, one should not visualize the expansion process as taking place into a surrounding vacuum or anything else: it is simply a continuous increase in distance between every pair of galaxies.

Note: Regarding spatial curvature, it is such that the three angles of a triangle add up to less than 180^o in an *open* universe ($k<0$) and to greater than 180^o in a *closed* universe ($k>0$). As mentioned above, a flat and an open universe expand forever, while a closed universe eventually contracts again and recollapses.

Q.20 Explain how *dark matter* can affect the large-scale geometry, thus also the evolution, of the Universe.

Ans. A key question in Cosmology is whether the density of matter in the Universe is as high as the critical density ρ_c needed to cause a recollapse ("Big Crunch") in the future (similar to a time-reversed Big Bang) rather than a continual expansion. The visible matter is much less than is needed to cause such a recollapse. Thus a central problem is determining how much dark matter there is in the Universe, and what it is made of. We note that ρ_c is about 10^{-26} kg/m^3, while the observed matter density is about 10^{-28} kg/m^3, much less than ρ_c . Current evidence seems to indicate that, despite the presence of much dark matter, the matter density present is less than ρ_c , which implies that the Universe will expand forever (*open universe*). This then means that the space-sections are of constant negative curvature (hyperbolic 3-space, spatially infinite universe). However, for theoretical reasons related to the "inflationary universe" idea, many astronomers believe that the Universe actually has almost-flat *spatial* sections ($k=0$), containing a large amount of yet undetected dark matter, much of which may be "exotic" (i.e. non-baryonic, unlike ordinary matter). (*Note:* It is the *spatial* sections that are flat; *spacetime* is still *curved* !)

Note that, according to a certain model, it is possible that the Universe be *spatially flat* and yet *spatially closed* (thus *finite*). On a large scale (i.e. globally), the Universe may have a torus-like topology, although locally (i.e., in the region we can observe) it may look like a Euclidean 3-space. To understand the situation, consider a 2-dimensional analogue: Topologically, a torus is a cylinder whose two end faces have been identified. A cylindrical surface is *intrinsically flat*, since it can be cut and developed onto a plane. The topology (i.e., the global connectivity) of a torus, however, is very different from that of an infinite plane (although *locally* they are both intrinsically flat).

Q.21 Describe the physical conditions leading to a *supernova* explosion of a star.

Ans. When a star of several solar masses evolves into a state of high density, the gravitational field can become so intense that the interior pressure is unable to support the weight of the outer layers of the star. The core of the star crushes itself in its own gravitational field and collapses on itself, until it reaches nuclear densities, where the collapse suddenly slows or stops. This sudden stop generates a shock wave that travels outward and rips off the outer layers of the star in a tremendous explosion, which we see as a supernova. Meanwhile, either the core attains a stable configuration as a *neutron star* or, if the core is too massive, it continues to collapse and ultimately forms a *black hole*.

Q.22 Explain the notion of *cosmological inflation*.

Ans. This hypothesis gives answers to a number of questions on the evolution of the Universe that the Big-Bang theory fails to address: (1) Why the density ρ of the Universe appears to be so close to the critical density ρ_c (i.e., the Universe is nearly flat). (2) Why the Universe is so uniform on large scales. For example, the CBR is uniform in temperature even over regions separated by vast distances. Such regions are so far apart that there has not been enough time since the Big Bang for even light to have traveled between them. (3) What is the ultimate origin of the relatively small-scale irregularities we see as galaxies.

According to the cosmological inflation hypothesis, there has been a period in the history of the Universe during which the latter underwent extremely rapid expansion, i.e. *inflation*. In a minute fraction of a second, the size of the Universe increased by 10^{30} times! Here are some of the conclusions of the theory:

1. Whatever the value of the density ρ of the Universe before inflation, afterwards it is extremely close to the critical value ρ_c . The rapid expansion causes *space* to become flatter in the same way the surface of an inflating balloon becomes flatter. A flat universe has $\rho = \rho_c$.

2. Regions which were thought to have always been far apart were actually very close just before inflation. So, it is quite reasonable that they have the same temperature after a period of inflation. This explains the uniform temperature of the CBR.

3. Quantum fluctuations in certain fields associated with inflation are a source of density fluctuations which ultimately lead to the formation of galaxies.

4. Inflation may explain why we do not see exotic objects like magnetic monopoles: the expansion dilutes them to unobservable levels.

Q.23 Explain the concept of the *cosmological constant*. What is the effect of the so-called *dark energy* in the expansion rate of the Universe?

Ans. The Einstein gravitational field equations are of the form $G_{\mu\nu} = \kappa T_{\mu\nu}$, where $G_{\mu\nu}$ is the symmetric Einstein tensor, built from second partial derivatives of the components of the metric tensor with respect to the various coordinates (this tensor describes the geometry of spacetime and becomes zero when spacetime is flat, i.e., has no curvature), $T_{\mu\nu}$ is the symmetric energy-momentum tensor describing the matter and energy responsible for the spacetime curvature, and κ is a gravitational constant. Einstein later inserted an extra term, adding $\Lambda g_{\mu\nu}$ to $G_{\mu\nu}$, where Λ is the "cosmological constant" (we assume $\Lambda>0$). This he did in order to allow the possibility of a static, unchanging universe as a particular solution of his equations. With the discovery of the expansion of the Universe (1929), however, he decided that $\Lambda=0$.

So, the Einstein static universe is possible only if $\Lambda\neq 0$. This term would then represent a universal *repulsive* force that balances the gravitational (attractive) forces which tend to make the Universe collapse, thus allowing a static situation. Generally, if $\Lambda \neq 0$, the expansion of the Universe must be faster than that which is predicted solely on the basis of the presence of matter (dark and visible), and there would be no chance of a universal recollapse. There are indications that this is indeed the case and, to account for this, modern Cosmology has postulated the existence of another as yet undetected substance which causes *anti-gravity* effects. This substance (whose nature is not known) is called the *dark energy*.

The cosmological term can be interpreted as an energy-momentum of the vacuum, i.e., an energy-momentum tensor not associated with matter but with empty (matter-free) space. If $\Lambda > 0$, the vacuum has an effective mass density $\rho_{eff} < 0$. The value of Λ may have been much larger during the early stages of the evolution of the Universe. Such a large value of Λ would have resulted in a fast expansion of the Universe (an *inflation*).

Q.24 Explain the role of the *weak interaction* in the nuclear processes that take place in the Sun (as well as in other stars).

Ans. The energy in the Sun comes from nuclear reactions involving the fusion of hydrogen to produce helium and heavier elements. However, for fusion to proceed according to the strong interaction (thus releasing large amounts of energy) the presence of *deuterium* (a proton-neutron system) is necessary. On the other hand, the nuclear reaction involving ordinary hydrogen (proton-proton reaction) proceeds very slowly via the *weak* interaction. Specifically, due to quantum mechanical effects, it is occasionally possible for two protons to overcome their electrical repulsion and come close enough to interact weakly. In this process, one of the protons beta decays to a neutron resulting in the formation of a deuterium nucleus. Then, proton-deuterium reactions are possible through the *strong* interaction, releasing large quantities of energy. This sequence of reactions is called the *proton-proton cycle*. Such reactions are believed to be the main source of the Sun's energy.

In many stars, however, where the temperatures are high enough, energy generation may proceed via *Bethe's carbon cycle*, which does not require a weak interaction as part of the process.

Bibliography

1. M. Alonso, E. J. Finn, *Physics* (Addison-Wesley, 1992).
2. H. Goldstein, *Classical Mechanics*, 2nd ed. (Addison-Wesley, 1980).
3. S. Hawking, *A Brief History of Time*, 2nd ed. (Bantam, 1996).
4. G. F. R. Ellis, R. M. Williams, *Flat and Curved Space-Times*, 2nd ed. (Oxford, 2000).
5. W. Rindler, *Relativity: Special, General and Cosmological* (Oxford, 2001).
6. H. C. Ohanian, R. Ruffini, *Gravitation and Spacetime*, 2nd ed. (Norton, 1994).
7. R. Hakim, *An Introduction to Relativistic Gravitation* (Cambridge, 1999).
8. P. J. Steinhardt, N. Turok, *Endless Universe: Beyond the Big Bang* (Doubleday, 2007).
9. G. D. Coughlan, J. E. Dodd, *The Ideas of Particle Physics*, 2nd ed. (Cambridge, 1991).
10. A. Hey, P. Walters, *The New Quantum Universe* (Cambridge, 2003).
11. B. C. Xanthopoulos, *About Stars and Universes* (Univ. of Crete, 1997) (*in Greek*).